Testataufgabe SW5
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501

a)

> $restart$

> $f := x \rightarrow x^3 + 2 \cdot x \cdot \sin(x)$

$$f := x \rightarrow x^3 + 2\,x \sin(x) \tag{1.1.1}$$

b)

> $f1 := D(f)$

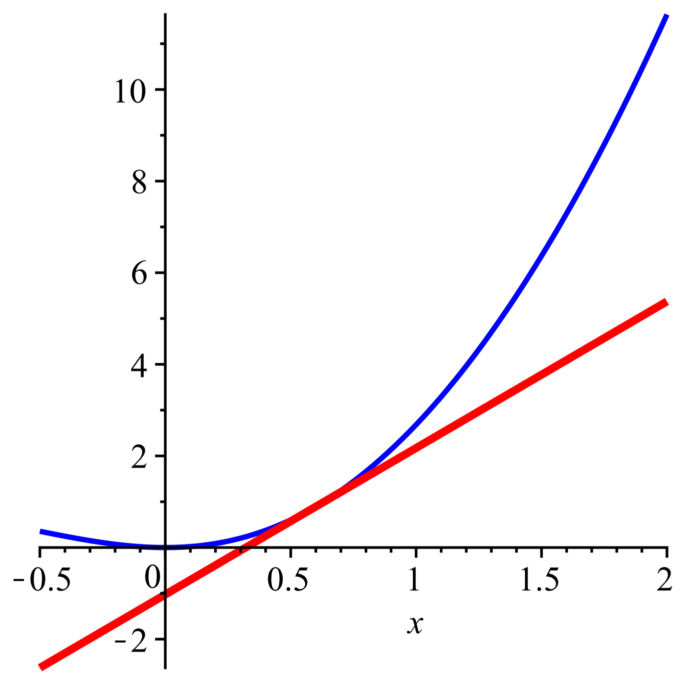
$$f1 := x \rightarrow 3\,x^2 + 2 \sin(x) + 2\,x \cos(x) \tag{1.2.1}$$

c)

> $t := x \rightarrow f(0.6) + f1(0.6) \cdot (x - 0.6)$

$$t := x \rightarrow f(0.6) + f1(0.6)\,(x - 0.6) \tag{1.3.1}$$

> $plot([f(x), t(x)], x = -0.5 ..2, color = [blue, red], thickness = [2, 3])$



502

> *restart*

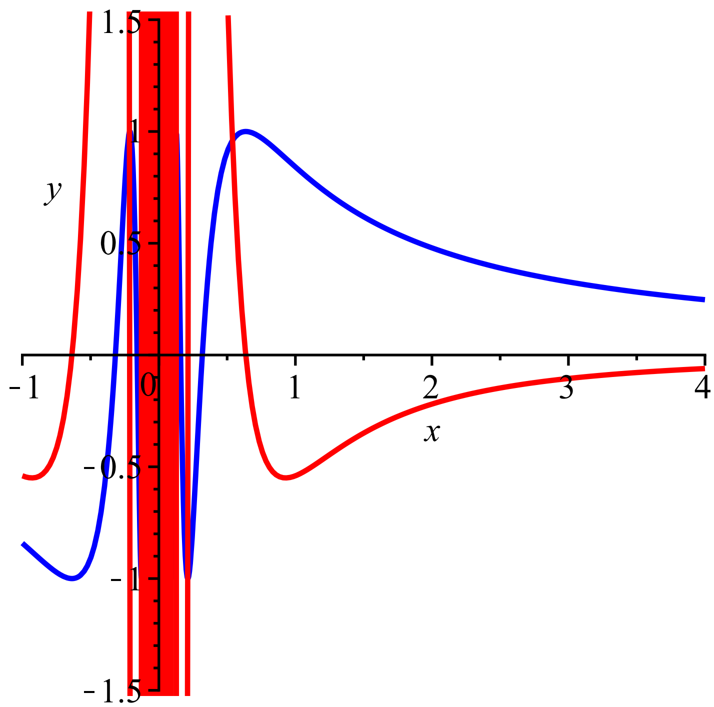
> $f := x \rightarrow \sin\left(\frac{1}{x}\right)$

$$f := x \rightarrow \sin\left(\frac{1}{x}\right) \qquad \textbf{(2.1)}$$

> $f1 := \mathrm{D}(f)$

$$f1 := x \rightarrow -\frac{\cos\left(\frac{1}{x}\right)}{x^2} \qquad \textbf{(2.2)}$$

> *plot*([$f(x)$, $f1(x)$], x = -1 ..4, y = -1.5 ..1.5, *color* = [*blue*, *red*], *thickness* = 2)



503

a)

> *restart*

> $Digits := 3$

$$Digits := 3 \quad (3.1.1)$$

> $f := x \to x \cdot (x-3) \cdot \exp(-x)$

$$f := x \to x\,(x-3)\,e^{-x} \quad (3.1.2)$$

> $f1 := D(f)$

$$f1 := x \to (x-3)\,e^{-x} + x\,e^{-x} - x\,(x-3)\,e^{-x} \quad (3.1.3)$$

> $xe := solve(f1(x) = 0, x)$

$$xe := \frac{5}{2} + \frac{1}{2}\sqrt{13}, \frac{5}{2} - \frac{1}{2}\sqrt{13} \quad (3.1.4)$$

> $xe2 := evalf(\%)$

$$xe2 := 4.30, 0.70 \quad (3.1.5)$$

>

b)

> $f2 := D(f1)$

$$f2 := x \to 2\,e^{-x} - 2\,(x-3)\,e^{-x} - 2\,x\,e^{-x} + x\,(x-3)\,e^{-x} \quad (3.2.1)$$

> $x1 := xe2[1]$

$$x1 := 4.30 \quad (3.2.2)$$

> $x2 := xe2[2]$

$$x2 := 0.70 \quad (3.2.3)$$

> $f2(x1)$

$$-0.0490 \quad (3.2.4)$$

> $y1 := f(x1)$

$$y1 := 0.0760 \quad (3.2.5)$$

Bei (x1, y1) ist ein lokales Maximum

> $f2(x2)$

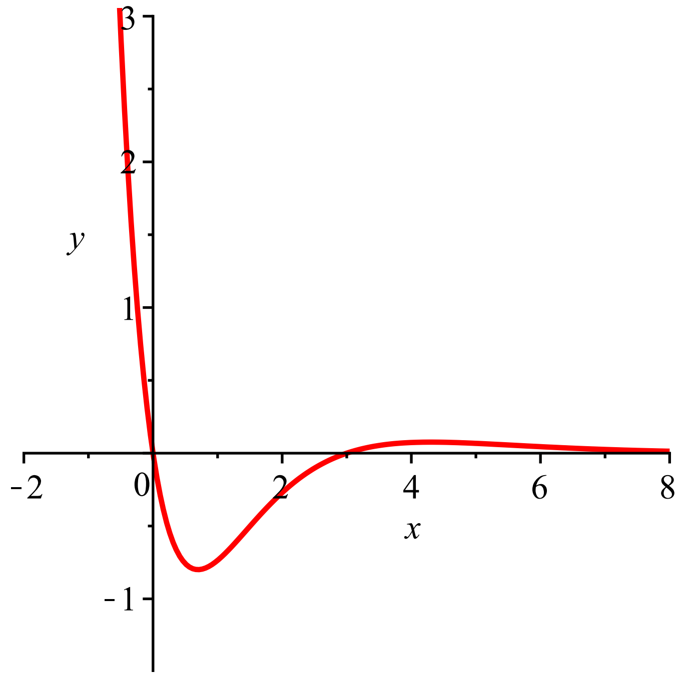
$$1.77 \quad (3.2.6)$$

> $y2 := f(x2)$

$$y2 := -0.800 \quad (3.2.7)$$

Bei (x2, y2) ist ein lokales Minimum

> $plot(f(x), x = -2..8, y = -1.5..3, thickness = 2)$



3
2
y
1
0
-1
-2
2
4
6
8
x

504

a)

> restart

> Digits := 3

$$Digits := 3 \quad \textbf{(4.1.1)}$$

> $f := x \rightarrow (x^2 - 1) \cdot \exp(-x)$

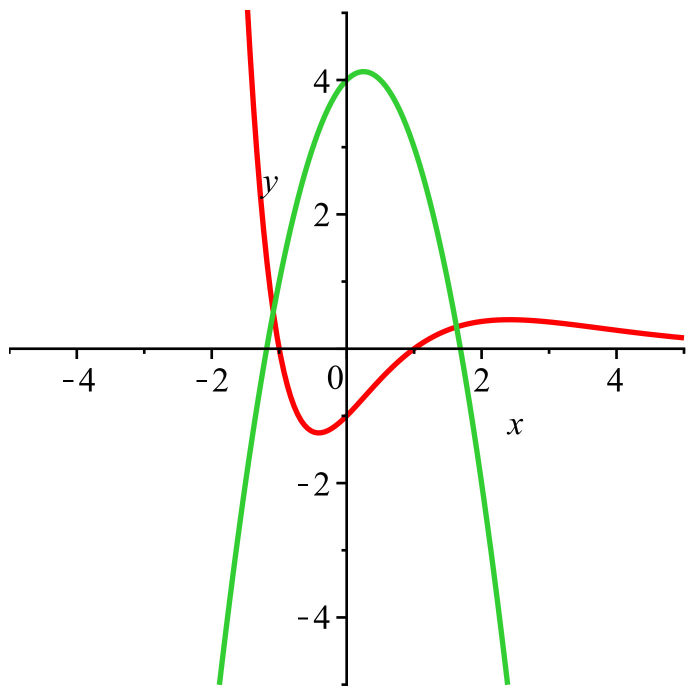
$$f := x \rightarrow (x^2 - 1)\, e^{-x} \quad \textbf{(4.1.2)}$$

> $g := x \rightarrow -2 \cdot x^2 + x + 4$

$$g := x \rightarrow -2\, x^2 + x + 4 \quad \textbf{(4.1.3)}$$

b)

> plot([f(x), g(x)], x = -5 ..5, y = -5 ..5, thickness = 2)



c)

1. Schnittpunkt:

> $x1 := fsolve(f(x) = g(x), x = -2 ..0)$

$$x1 := -1.09 \quad \textbf{(4.3.1)}$$

> $y1 := f(x1)$

$$y1 := 0.564 \quad \textbf{(4.3.2)}$$

2. Schnittpunkt:

> $x2 := fsolve(f(x) = g(x), x = 0.5 ..2)$

$$x2 := 1.63 \quad \textbf{(4.3.3)}$$

> $y2 := f(x2)$

(4.3.4)

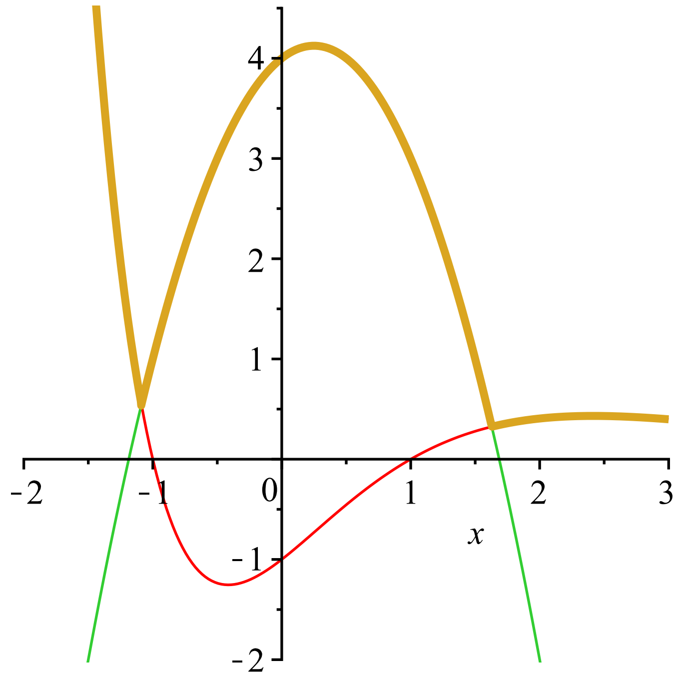
$$y2 := 0.325 \qquad (4.3.4)$$

d)

> $h := x \rightarrow piecewise(x < x1, f(x), x < x2, g(x), f(x))$

$$h := x \rightarrow piecewise(x < x1, f(x), x < x2, g(x), f(x)) \qquad (4.4.1)$$

> $plot([f(x), g(x), h(x)], x = -2 ..3, -2 ..4.5, thickness = [1, 1, 3])$



e)

> $\int_{x1}^{x2} g(x)\, dx - \int_{x1}^{x2} f(x)\, dx$

$$9.19 \qquad (4.5.1)$$

505

a)

> *restart*

> *Digits* := 3

$$Digits := 3 \qquad (5.1.1)$$

> $f := x \to a \cdot x^3 + b \cdot x^2 + c \cdot x + d$

$$f := x \to a\,x^3 + b\,x^2 + c\,x + d \qquad (5.1.2)$$

> $gl := \{f(-1) = 14.4, f(0) = 10, f(1) = 9.6, f(2) = 19.8\}$

$$gl := \{d = 10, -a + b - c + d = 14.4, a + b + c + d = 9.6, 8\,a + 4\,b + 2\,c + d = 19.8\} \qquad (5.1.3)$$

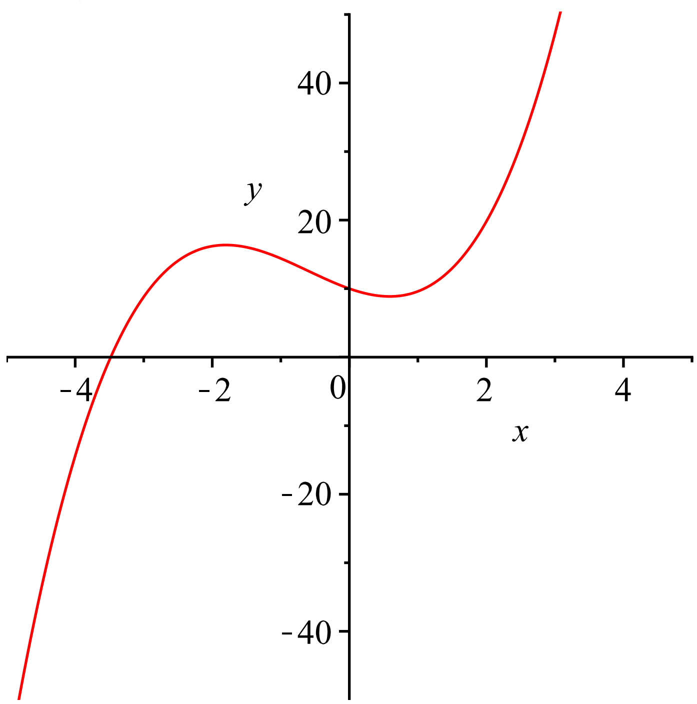
> *solve*(*gl*, {*a*, *b*, *c*, *d*})

$$\{a = 1.10, b = 2., c = -3.50, d = 10.\} \qquad (5.1.4)$$

> *assign*(%)

b)

> *plot*(*f*(*x*), *x* = -5 ..5, *y* = -50 ..50)



> *fsolve*(*f*(*x*) = 0)

$$-3.48 \qquad (5.2.1)$$

c)

> *f1* := D(*f*)

$$f1 := x \to 3\,a\,x^2 + 2\,b\,x + c \qquad (5.3.1)$$

> *xe* := *solve*(*f1*(*x*) = 0, *x*)

$$xe := 0.589, -1.80 \qquad (5.3.2)$$

> $f2 := D(f1)$

$$f2 := x \rightarrow 6\,a\,x + 2\,b \qquad \textbf{(5.3.3)}$$

> $f2(x1)$

$$6.60\,x1 + 4. \qquad \textbf{(5.3.4)}$$

> $f2(x2)$

$$6.60\,x2 + 4. \qquad \textbf{(5.3.5)}$$

> $x1 := xe[1]; x2 := xe[2];$

$$x1 := 0.589$$
$$x2 := -1.80 \qquad \textbf{(5.3.6)}$$

> $y1 := f(x1); y2 := f(x2);$

$$y1 := 8.86$$
$$y2 := 16.4 \qquad \textbf{(5.3.7)}$$

Bei (x1, y1) ist es ein lokales Minimum.
Bei (x2, y2) ist es ein lokales Maximum.