Differentialrechnung – Übungen II

SW10

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1) $y = x^3 - 4x$ & Schnittpunkt x-/y-Achse?

x-Achse: $y = 0$

$0 = x^3 - 4x$

$0 = x(x^2 - 4)$ $\quad x_1 = 0$

$0 = x^2 - 4$

$x = \sqrt{4}$ $\quad x_2 = \pm 2$

$f'(x) = [x^3 - 4x]'$

$= 3x^2 - 4$

$x_1 \Rightarrow -4 \Rightarrow \arctan(-4) = -75{,}96°$ ✓

$90° - 75{,}96° = 14{,}04°$

$x_2 = 3(2)^2 - 4$

$= 8$

-4

$\arctan(8) = 82{,}87°$

$\arctan(-8) + 90° = 7{,}13°$

3) $y_1 = ax^3$ $\quad a = ?$ $\quad y_1' = 3ax^2$

$y_2 = 6x + 4$ $\quad y_2' = 6$

$y_1 = y_2 \Rightarrow ax^3 = 6x + 4$

$y_1' = y_2' \Rightarrow 3ax^2 = 6$

$ax^2 = 2$

$ax^3 = 2x \Rightarrow 2x = 6x + 4$

$4x + 4 = 0$

$x = -1$

$y = ax^3 = 2$

$a(-1)^3 = 2$

$a = -2$

$a = 2 : P(-1, -2)$ ✓

4) a) $f: \mathbb{R} \to \mathbb{R}$, $x \to f(x) = \frac{1}{2} - 3$

$x = \frac{1}{2} - 3$

$\frac{1}{2} = x + 3$

$y = 2x + 6$

b) $g: (-1, 0) \to \mathbb{R}^+$, $x \to g(x) = \frac{x}{x^2 - 1}$

4c?

5) a) $f(x) = \frac{\sqrt{x}}{x^2 \sqrt[3]{x}}$

$= \frac{x^{\frac{1}{2}}}{x^2 \, x^{\frac{1}{3}}}$

$= \frac{x^{\frac{1}{2}}}{x^{\frac{7}{3}}}$

$= x^{\frac{3}{6} - \frac{14}{6}}$

$= x^{-\frac{11}{6}}$

$f'(x) = \left[x^{-\frac{11}{6}}\right]'$

$= -\frac{11}{6} \cdot x^{-17/6}$ ✓

b) $f(x) = \sqrt[3]{\frac{x^3 \sqrt{x}}{a}}$

$= \frac{x \cdot x^{1/6}}{a^{1/3}} = \frac{x^{7/6}}{a^{1/3}}$

$f'(x) = \left[\frac{x^{7/6}}{a^{1/3}}\right]'$

$= \frac{7}{6} \cdot \left(\frac{x^{1/6}}{a^{1/3}}\right)$

$= \frac{7}{6} \cdot \sqrt[6]{x/a^2}$

6) $y_1 = 1/x$ $\quad f'(x) = \tan(\alpha)$

$y_2 = \sqrt{x^2 - 2}$

$\frac{1}{x} = \sqrt{x^2 - 2}$

$0 = \sqrt{x^2 - 2} - 1/x$

$x_{1/2} = \pm \sqrt{1 - \sqrt{2}}$ $\quad <0$ nicht möglich

$x = \sqrt{1 - \sqrt{2}}$

$f'(y_1) = \left[\frac{1}{x}\right]'$

$= \frac{[1]' \cdot x - 1 \cdot [x]'}{x^2}$

$= \frac{0 \cdot x - 1 \cdot 1}{x^2}$

$= \frac{-1}{x^2}$

$f'(y_2) = \left[(x^2 - 2)^{1/2}\right]'$

$= \frac{1}{2} \cdot (x^2 - 2)^{-1/2} \cdot (2x - 0)$

$= \frac{x}{\sqrt{x^2 - 2}}$

$f'(x) = \tan(\alpha)$

y_1: $\alpha = -\tan^{-1}(1/x^2) = 67{,}5°$

y_2: $\alpha = \tan^{-1}\left(\frac{x}{\sqrt{x^2-2}}\right) = -22{,}5°$

$180° - 67{,}5° - 22{,}5° = 90°$ ✓